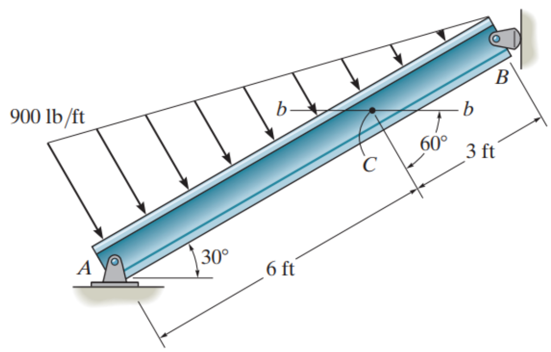
Problem 1-3

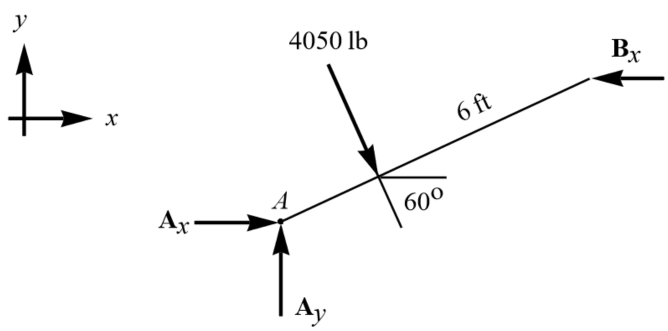
Determine the resultant internal loadings acting on section b–b through the centroid C on the beam.



Prob. 1–3

Solution

The smooth surface at B prevents horizontal motion, and the external pin at A prevents horizontal and vertical motion. As a result, there are three unknown forces in the free-body diagram of the beam shown below.



The distributed force is treated as a single resultant force applied through the centroid of the triangle,

$$\frac{2}{3}(9 \text{ ft}) = 6 \text{ ft},$$

with a magnitude given by the area of the triangle,

$$A = \frac{1}{2}\left(900 \frac{\text{lb}}{\text{ft}}\right)(9 \text{ ft}) = 4050 \text{ lb}.$$

Use the equations of equilibrium to determine the reaction forces, taking the sum of the moments about point A.

$$\sum F_x = A_x - B_x + 4050 \cos 60^\circ = 0$$

$$\sum F_y = A_y - 4050 \sin 60^\circ = 0$$

$$\circlearrowleft^{+} \sum M_A = -(4050 \text{ lb})(3 \text{ ft}) + B_x(9 \sin 30^\circ \text{ ft}) = 0$$

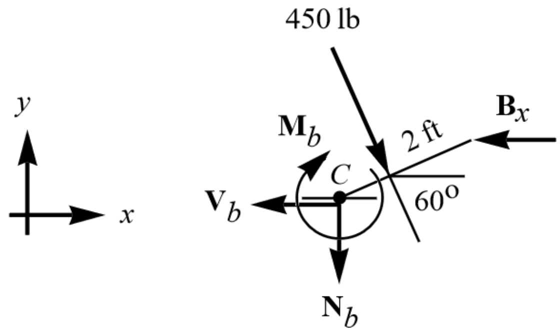
Solving this system of equations yields

$$A_x = 675 \text{ lb}$$

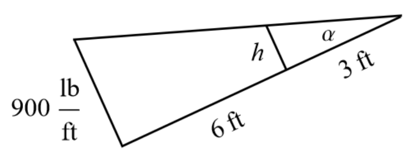
$$A_y = 2025 \text{ lb}$$

$$B_x = 2700 \text{ lb}.$$

Now that the reactions at A and B are known, the internal loadings on section b–b through point C can be determined using the method of sections. Since it's simpler, choose the section of the beam from B to C.



This 450-lb force is the resultant force of the part of the distributed force from B to C. The height of the distributed force triangle at C is found using similar triangles.



$$\tan\alpha = \frac{900}{9} = \frac{h}{3} \quad \rightarrow \quad h = 300 \ \frac{\text{lb}}{\text{ft}}$$

As a result, the part of the distributed force from B to C has an area of

$$A = \frac{1}{2}\left(300 \ \frac{\text{lb}}{\text{ft}}\right)(3 \text{ ft}) = 450 \text{ lb}.$$

This is the magnitude of the force; it passes through the centroid, which is

$$\frac{2}{3}(3 \text{ ft}) = 2 \text{ ft}$$

from B. Use the equations of equilibrium to find the internal loadings at C.

$$\sum F_x = -V_b - B_x + 450\cos 60^\circ = 0$$

$$\sum F_y = -N_b - 450\sin 60^\circ = 0$$

$$\circlearrowleft^{+} \sum M_C = -M_b - (450 \text{ lb})(1 \text{ ft}) + B_x(3\sin 30^\circ \text{ ft}) = 0$$

Solving this system of equations yields

$$V_b = 225 - B_x = -2475 \text{ lb}$$

$$N_b = -225\sqrt{3} \approx -390 \text{ lb}$$

$$M_b = \frac{3}{2}(-300 + B_x) \approx 3600 \text{ lb} \cdot \text{ft}.$$

A minus sign means that the vector is in the opposite sense than indicated in the free-body diagram.